647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2023 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer any two **[10]**

(1) In usual notation prove that $(a * b) * c = a * (b * c)$ and $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

(2) State and prove Distributive inequalities.

(3) Define Complement element. $(L, *, \oplus, 0, 1)$ is distributive lattice, then prove that complement element of $a \in L$ exist than it is unique.

Q.1(B) Answer any one **[04]**

(1) In usual notation show that (S_{30}, D) is poset.

(2) Let $(L, \leq)$ be a Lattice, in usual notation prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

Q.2(A) Answer any two **[10]**

(1) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $a, b \in B$ then prove that

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$.

(2) In usual notation for Boolean algebra B, $x_1, x_2 \in B$ prove that $A(x_1 * x_2) = A(x_1) \cap A(x_2)$.

(3) Obtain sum of product canonical form of $\alpha(x_1, x_2, x_3) = x_2 \oplus x_3$

Q.2(B) Answer any one **[04]**

(1) For Boolean statement prove that $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c') = b' * (a \oplus c)$

(2) For a Boolean algebra B, prove that $a \neq 0$ is an atom in B if and only if for any $x \in B$, $a * x = 0$ or $a * x = a$.

Q.3(A) Answer any two **[10]**

(1) In usual notation obtain Cauchy – Riemann conditions in Cartesian form.

(2) Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$ satisfies Laplace equation. However $f(z) = u + iv$ is not an analytic function.

(3) Prove that $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$

$f(z) = 0$, $z = 0$ satisfies C – R conditions at origin however f(z) is not analytic at origin.

Q.3(B) Answer any one **[04]**

(1) Show that $\frac{y}{x^2 + y^2}$ is a harmonic function and find it's conjugate harmonic function.

(2) Obtain Laplace equation in polar form.

Q.4(A) Answer any two. **[10]**

(1) State and prove Cauchy's fundamental theorem for integration.

(2) Find $\int_c \frac{z}{(9 - z^2)(z + i)} dz$; where C: $|z| = 2$.

(3) Show that $\left|\int_c \frac{\log z}{z^2} dz\right| \leq 2\pi \frac{(\pi + \log R)}{R}$ Where C be the circle $|z| = R$ (R > 1) described in the counter clock Wise direction.

Q.4(B) Answer any one. **[04]**

(1) In usual notation prove that $\left|\int_c f(z)dz\right| \leq ML$.

(2) Find: $\int_c \frac{(5z - 2)dz}{z(z-3)}$ where C: $|z + 1| = \frac{3}{2}$.

Q.5(A) Answer any two **[10]**

(1) State and prove the fundamental theorem of algebra.

(2) State and prove Liouville's theorem.

(3) State and prove Morera's theorem.

Q.5(B) Answer any one **[04]**

(1) Find: $\int_c \frac{e^{2z}}{(z-1)^4}$ dz where C: $|z| = 2$.

(2) Prove that $\int_c \frac{e^{kz}}{z}$ dz $= 2\pi i$, C: $|z| = 1$ and hence deduce that

(i) $\int_0^{\pi} e^{k\cos\theta} \cos(k\sin\theta)d\theta = \pi$ **(ii)** $\int_{-\pi}^{\pi} e^{k\cos\theta} \sin(k\sin\theta)d\theta = 0$

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